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THERMAL RADIATION AND CONVECTIVE HEATING ON HYDROMAGNETIC BOUNDARY LAYER FLOW OF NANOFLUID PAST A PERMEABLE STRETCHING SURFACE

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ABSTRACT

This research work studies the hydromagnetic boundary layer flow of Nanofluid past a permeable stretching surface with the introduction of both thermal radiation and Newtonian heating. The Nanoparticles considered here are Copper (Cu) and Alumina (Al_2O_3) while water served as the base fluid. The derived dimensionless governing equations for this investigation are solved using a set of codes on the MAPLE software. The effects of significant physical parameters on velocity, temperature, skin friction and Nusselt number profiles within the boundary layer of the two water-based Nanofluids are investigated with interpretations from the graphs.

KEYWORDS*: Boundary Layer, Thermal Radiation, Nanofluid, Stretching Surface*

Article History

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INTRODUCTION

Nanofluids are dilute liquid suspensions of Nanoparticles with at least one of their principal dimensions smaller than 100nm and have been found to possess enhanced thermophysical properties such as thermal conductivity, thermal diffusivity, viscosity, and convective heat transfer coefficients compared to those of base fluids like oil or water. Boundary layer flow and heat transfer of a fluid over a stretching surface have been a major focus for myriads of researchers in recent years. It is discovered to have a large range of applications in many manufacturing fields, such as manufacturing process of artificial fibers, artificial films and dilute polymer solutions, modern metallurgy, and metal-working processes. The analysis of heat transfer over a stretching surface is of great practical interest because of its wide applications which include materials manufactured by extrusion (i.e polymer extrusion), paper production, glass fiber production, crystal growing, cooling of metallic sheets or electronic chips, drawing of liquid and plastic films in condensation processes, copper wire drawing. In view of these applications, theoretical study of Boundary layer flow over the stretching surface was pioneered by O. D Makinde [2]. Thereafter, various categories of fluid flows and heat transfer problems for stretching surfaces have been explored in series of investigations (see for instance [6, 7, 16, 13]). A. Postelnicu [3], W. A Khan [1, 4]. Makinde [5] investigated the inherent irreversibility in hydromagnetic boundary layer flow of variable viscosity fluid over a semi-infinite flat plate under the influence of thermal radiation and Newtonian heating.

The effect of thermal radiation and viscous dissipation on boundary layer flow of Nanofluid over a permeable moving flat plate was analyzed by T. G. Motsumi and O. D Makinde [8]. Ghara *et. al*. [9] analyzed the effects of radiation on MHD free convection flow past an impulsively moving vertical plate with ramped wall temperature. Keshtkar and Amiri [10] examined MHD steady flow and heat transfer of an incompressible Nanofluid over a non-linearly stretching and permeable sheet. Kishan and Kavitha [12] analyzed the Non-Newtonian magneto-hydro dynamic boundary layer flow of an electrically conducting power-law fluid flowing over a non-linear stretching surface in the presence of thermal radiation, considering the viscous dissipation effects. Haile and Shankar [14] considered a steady boundary layer flow of water-based Nanofluid over a moving Permeable surface and the plate was taken to move in the same or opposite direction to the free stream. Zafariyan and Fanaee [11] investigated the effects of thermal radiation on steady MHD mixed convection over a vertical plate with a convective boundary condition embedded in a porous medium.

In this paper, our main objective is to analyze the combined effects of thermal radiation and Newtonian heating on hydromagnetic boundary layer flow of Nanofluid past a permeable stretching surface with the introduction of both thermal radiation and Newtonian heating. The nanoparticles considered are Copper (Cu) and Alumina (Al_2O_3) while water served as the base fluid. The derived dimensionless governing equations are solved using MAPLE software. The effects of various significant physical parameters on velocity, temperature, skin friction and Nusselt number profiles within the boundary layer of the two water-based Nanofluids are presented graphically.

NOMENCLATURE

(u, v)	velocity components along x and y-direction
U_0	plate velocity
T	temperature of the nanofluid
T_f	temperature of the hot convectional fluid
μ_{nf}	dynamic viscosity of the nanofluid
ρ_{nf}	density of the nanofluid
σ_{nf}	electrical conductivity of the nanofluid
α_{nf}	thermal diffusivity of the nanofluid
$(\rho C_p)_{nf}$	heat capacitance of the nanofluid
K_{nf}	thermal conductivity of the nanofluid
V_{nf}	kinematic viscosity of the nanofluid
q_r	radiative heat flux

MATHEMATICAL FORMULATION

A steady unidirectional boundary layer flow of an electrically conducting Nanofluid past a permeable stretching surface in the presence of a uniform transverse magnetic field whose strength B_o applied parallel to the y-axis as shown in figure 2 was studied. We take the effects of the induced magnetic field and the external electric field as inconsequential. At the boundary, the permeable plate is moving at a velocity U_0 with a hot convectional fluid of temperature T_f flowing below it and a cold Nanofluid of temperature $T < T_f$ flowing above the plate. Far away from the plate, $u = 0$, $T=T_\infty$. We take x-axis along the direction of plate and y-axis normal to it. The surface temperature is assumed to be kept constant by convective heat transfer at a constant temperature T_f. Under the boundary-layer approximation, the Nanofluid equations for continuity, momentum and energy balance governing the problem under consideration in one dimension are written as

$$\frac{\partial x}{\partial y} = 0 \tag{1}$$

$$-V\frac{du}{dy} = -\frac{1}{\rho_{nf}}\frac{dp}{dx} + \frac{\mu_{nf}}{\rho_{nf}}\frac{d^2u}{dy^2} - \frac{\sigma_{nf}B_0^2u}{\rho_{nf}} - \frac{V_{nf}}{k}u \tag{2}$$

$$-V\frac{dT}{dy} = \frac{K_{nf}}{(\rho C_p)_{nf}}\frac{d^2T}{dy^2} + \frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{du}{dy}\right)^2 + \frac{\sigma_{nf}B_0^2u}{(\rho C_p)_{nf}} - \frac{1}{(\rho C_p)_{nf}}\left(\frac{dq_r}{dy}\right) \tag{3}$$

With the boundary conditions

$$u(x,0) = U_0, -K_f\frac{dT}{dy}(-x,0) = h_f(T_f - T(x,0)) \tag{4}$$

$$u(x,0) = 0, T(x,0) = T_\infty \tag{5}$$

$$u = U_0, v = V \ - K_f\frac{dT}{dy} = h_f(T_f - T) \tag{6}$$

Introducing the following dimensionless variables and quantities into the above equation

$$\eta = \frac{yU_\circ}{V_f},\ W = \frac{u}{U_\circ},\ S = \frac{V}{U_\circ},\ \theta = \frac{T - T_\infty}{T_f - T_\infty},\ V_f = \frac{\mu_f}{\rho_f},\ \Pr = \frac{V_f}{\alpha_f},\ Bi = \frac{hV_f}{U_\circ k_f},\ \alpha_f = \frac{k_f}{(\rho C_p)_f} \tag{7}$$

$$Ha = \frac{\sigma_f B_\circ^2 V_f}{\rho_f U_\circ^2},\ Br = \frac{\mu_f U_\circ^2}{k_f(T_f - T_\infty)},\ \mu_{\eta f} = \frac{\mu_f}{(1-\varphi)^{2.5}},\ V_{\eta f} = \frac{\mu_{\eta f}}{\rho_{\eta f}},\ \rho_{\eta f} = (1-\varphi)\rho_f + \varphi\rho_s, \tag{8}$$

$$\rho_{\eta f} = (1-\varphi)\sigma_f + \varphi\sigma_s,\ (\rho C_p)_{\eta f} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s,\ \sigma_{\eta f} = (1-\varphi)\sigma_f + \varphi\sigma_s, \tag{9}$$

$$\frac{K_{nf}}{K_f} = \frac{(k_s + 2k_f) - 2\varphi(k_f - k_s)}{((k_s + 2k_f) - \varphi(k_f - k_s))}, \tag{10}$$

$$q_r = -\frac{4\sigma}{3\kappa}\frac{\partial T^4}{\partial y} \tag{11}$$

$$\frac{d^2W}{d\eta^2} + S(1-\varphi)^{2.5}\left[(1-\varphi) + \varphi\frac{\rho_s}{\rho_f}\right]\frac{dW}{d\eta} - (1-\varphi)^{2.5} Ha\left[(1-\varphi) + \varphi\frac{\sigma_s}{\sigma_f}\right]W - \frac{1}{\kappa}W = 0 \tag{12}$$

$$\left(1+\frac{4}{3N}\left(\frac{(k_s+2k_{nf})+\phi(k_f-k_s)}{(k_s+2k_f)-2\phi(k_f-k_s)}\right)\right)\frac{d^2\theta}{d\eta^2}+S\Pr\left(\frac{(k_s+2k_{nf})+\phi(k_f-k_s)}{(k_s+2k_f)-2\phi(k_f-k_s)}\right)\left[(1-\phi)+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right]\frac{d\theta}{d\eta}-$$

$$\cdot\left(\frac{(k_s+2k_{nf})+\phi(k_f-k_s)}{(k_s+2k_f)-2\phi(k_f-k_s)}\right)\frac{Br}{(1-\phi)^{2.5}}\left(\frac{dW}{d\eta}\right)^2+\left(\frac{(k_s+2k_{nf})+\phi(k_f-k_s)}{(k_s+2k_f)-2\phi(k_f-k_s)}\right)\left[(1-\phi)+\phi\frac{\sigma_s}{\sigma_f}\right]HaBrW^2=0$$

with the boundary conditions:

$$W=1,\frac{d\theta}{d\eta}=Bi(\theta-1)\ \text{at}\ \eta=0$$

$$W=0,\theta=0\ \text{as}\ \eta\to\text{infinity}$$

For skin Friction and Nusselt Number

$$C_f=\frac{\tau_w}{\rho_f U_0^2}\ \text{becomes}\ \frac{1}{(1-\phi)^{2.5}}\frac{dW}{d\eta},\quad Nu=\frac{xq_w}{k_f(T_f-T_\infty)}\ \text{also becomes}\ Nu=-\frac{k_{nf}}{k_f}\operatorname{Re}x\frac{d\theta}{d\eta}$$

RESULTS AND DISCUSSIONS

The derived dimensionless governing equations are solved using MAPLE software. The effects of significant physical parameters on velocity, temperature, skin friction and Nusselt number profiles within the boundary layer of the two water-based Nanofluids are investigated with interpretations from the graphs. The thermophysical properties of water and Nanoparticles are presented in the table below.

Table 1: Thermo Physical Properties of Water and Nanoparticles

Materials	$\rho(kg/m^3)$	$C_p(J/kgK)$	$k(W/mK)$	$\sigma(S/m)$
Pure water	997.1	4179	0.613	5.5×10^{-6}
Copper (Cu)	8933	385	400	58×10^{6}
Alumina (Al_2O_3)	3970	765	40	35×10^{6}

RESULTS

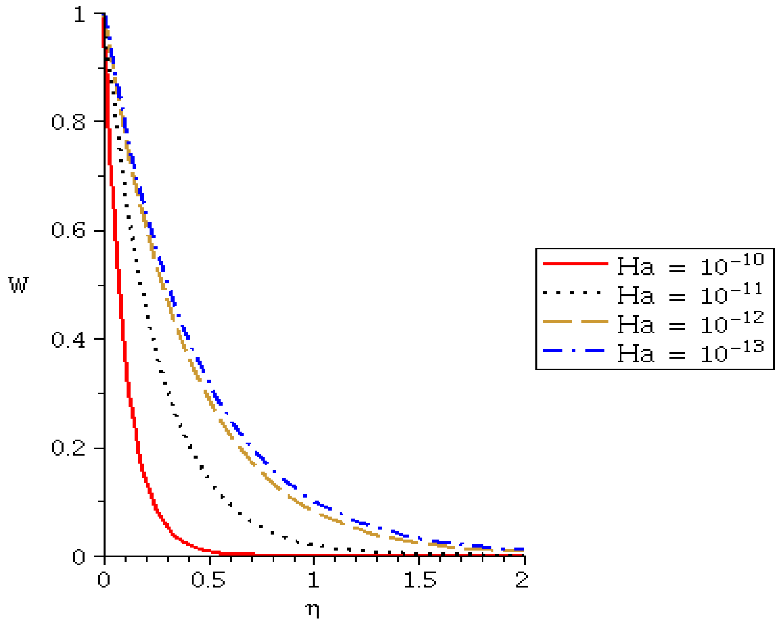

Figure 1: Velocity Profile with Decreasing Hartmann Number

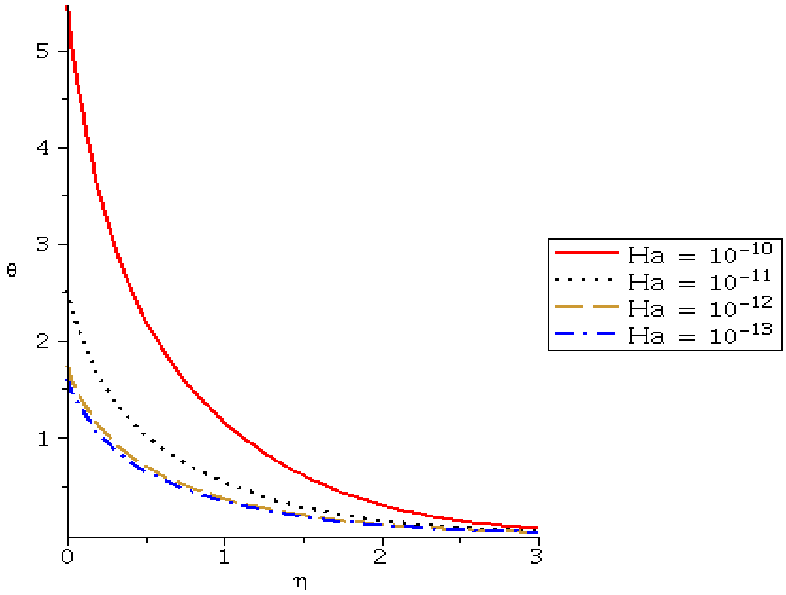

Figure 2: Temperature Profile with Decreasing Hartmann Number

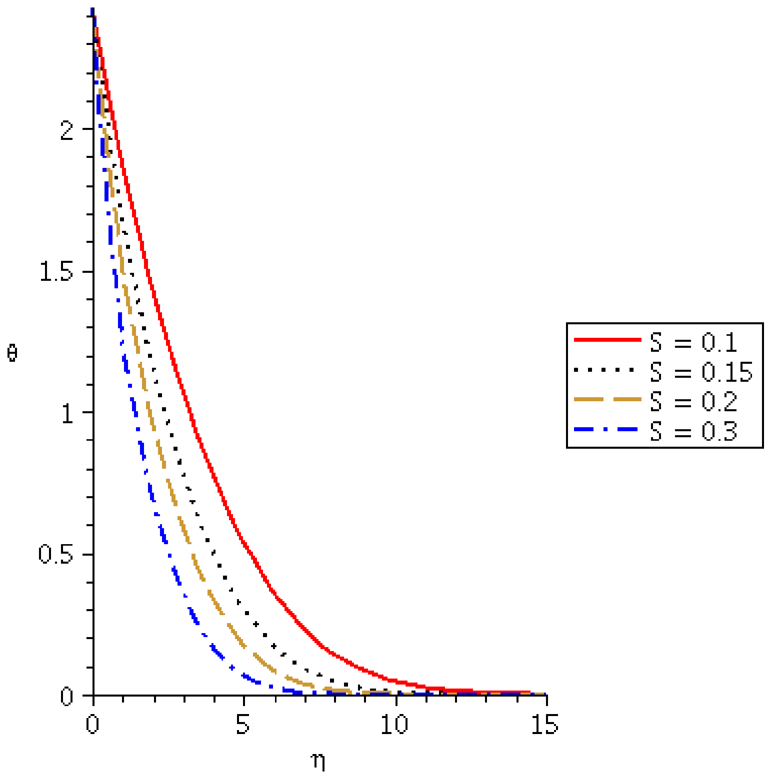

Figure 3: Temperature Profile with Increasing Suction Parameter

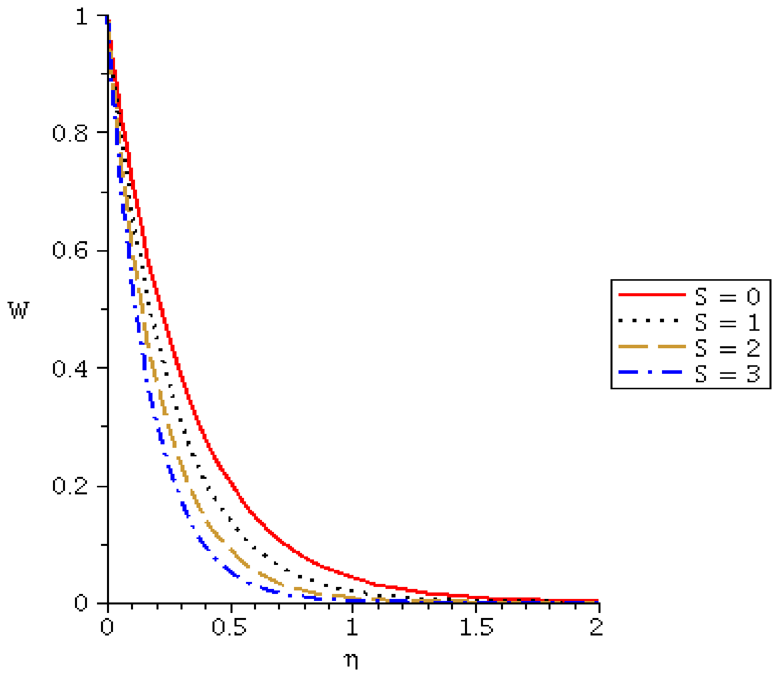

Figure 4: Velocity Profile with Increasing Suction Parameter

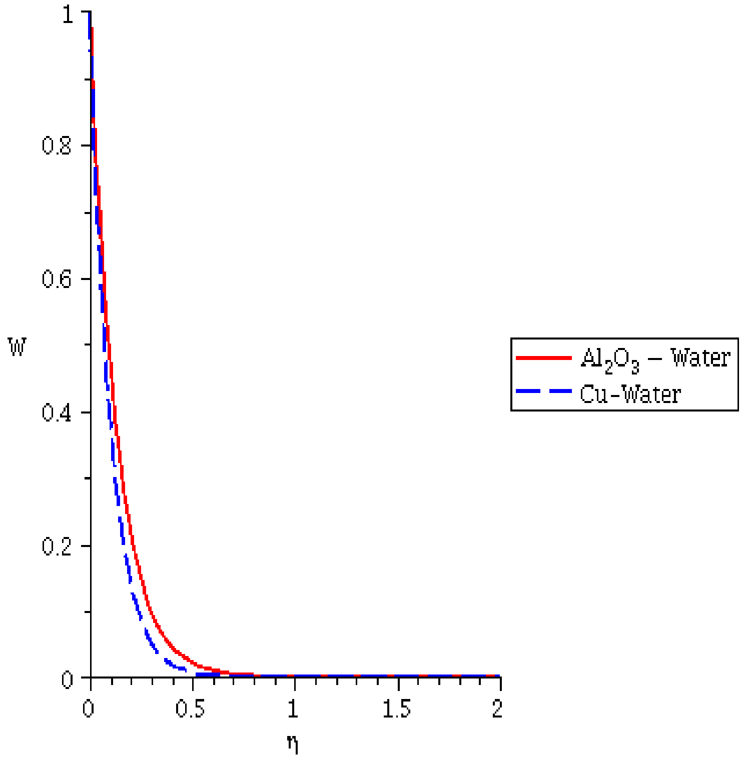

Figure 5: Velocity Profile for Different Nanofluid

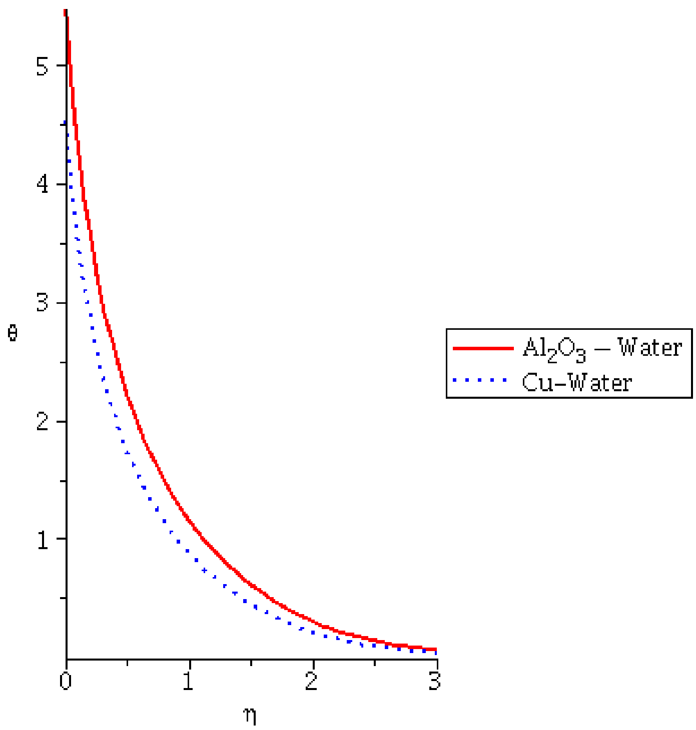

Figure 6: Temperature Profile for Different Nanofluid

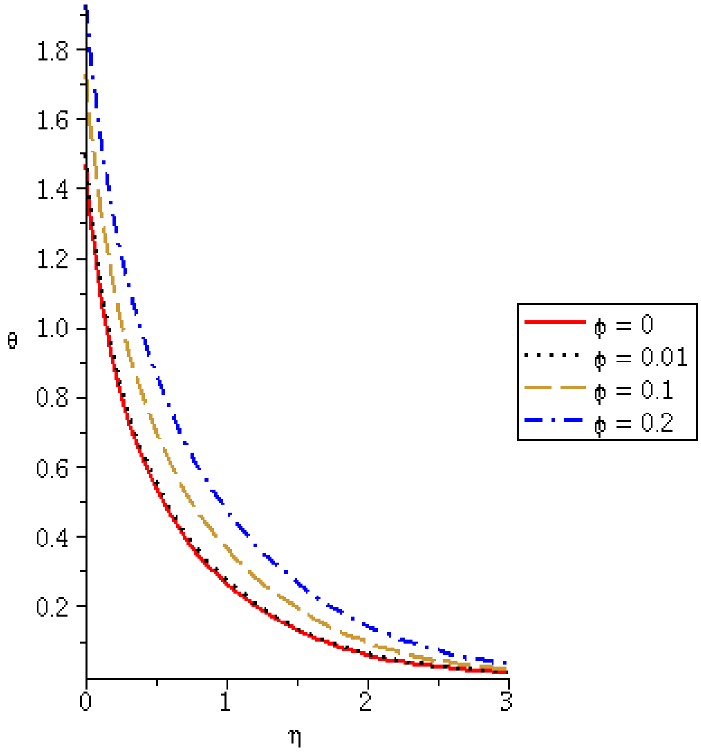

Figure 7: Temperature Profile with Increasing Solid Volume Fraction Parameter

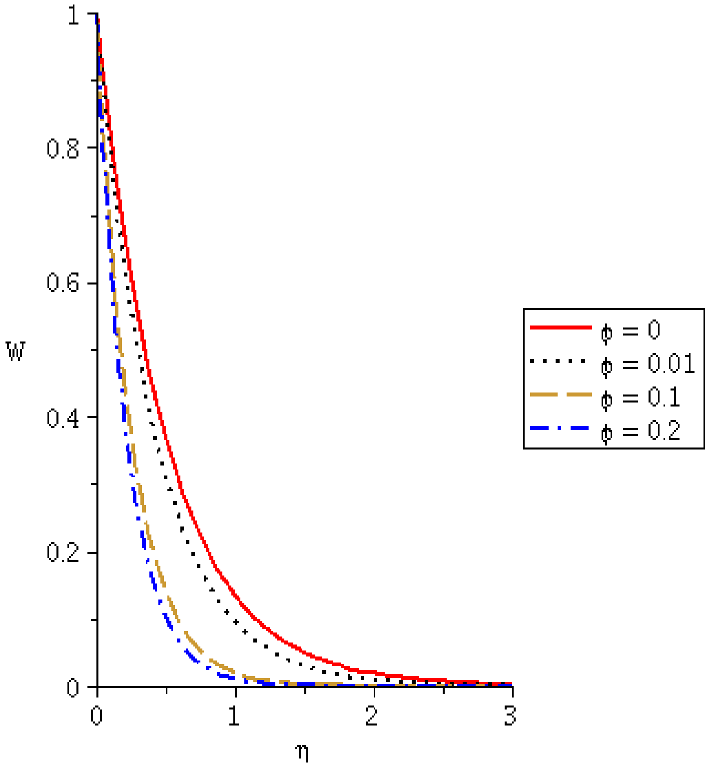

Figure 8: Velocity Profile with Increasing Solid Volume Fraction Parameter

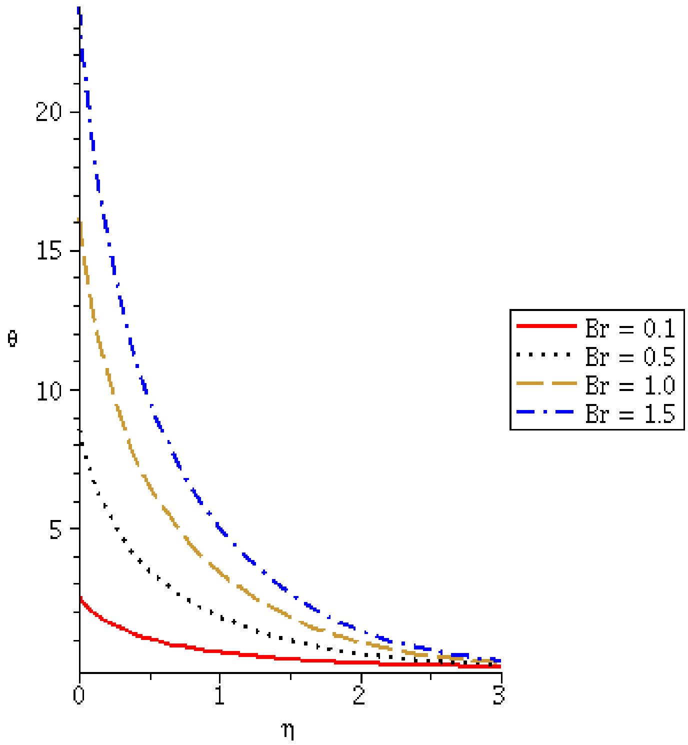

Figure 9: Temperature Profile with Increasing Brinkmann Number

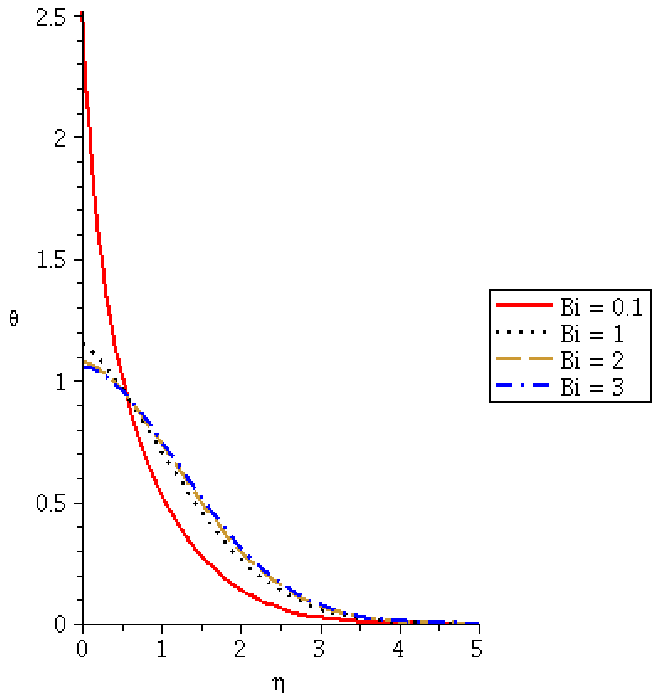

Figure 10: Temperature Profile with Increasing Biot Number

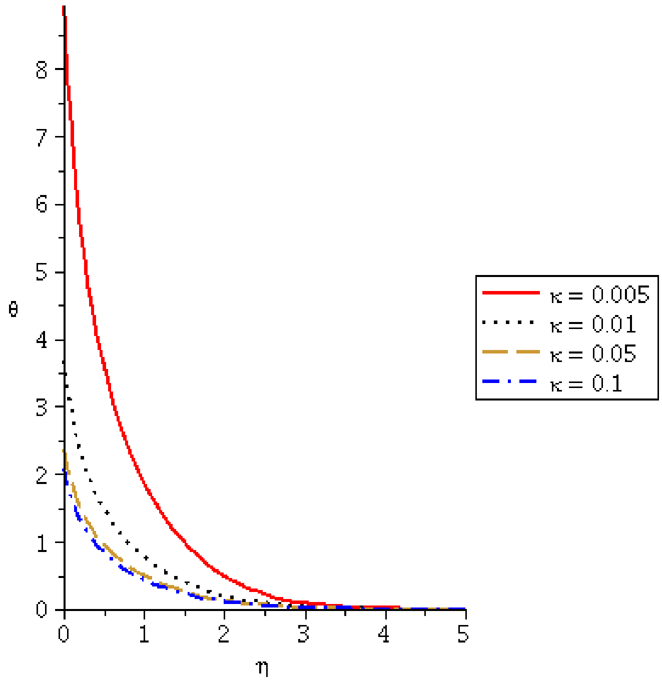

Figure 11: Temperature Profile with Increasing Permeability Parameter

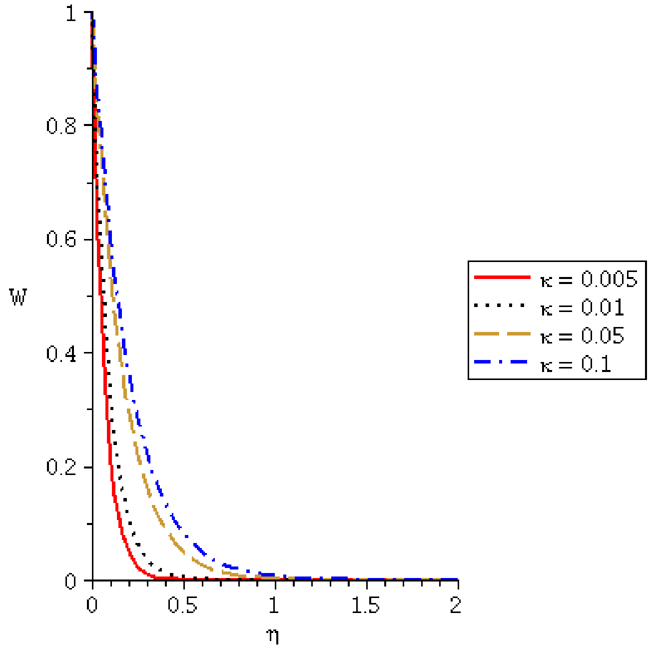

Figure 12: Velocity Profile with Increasing Permeability Parameter

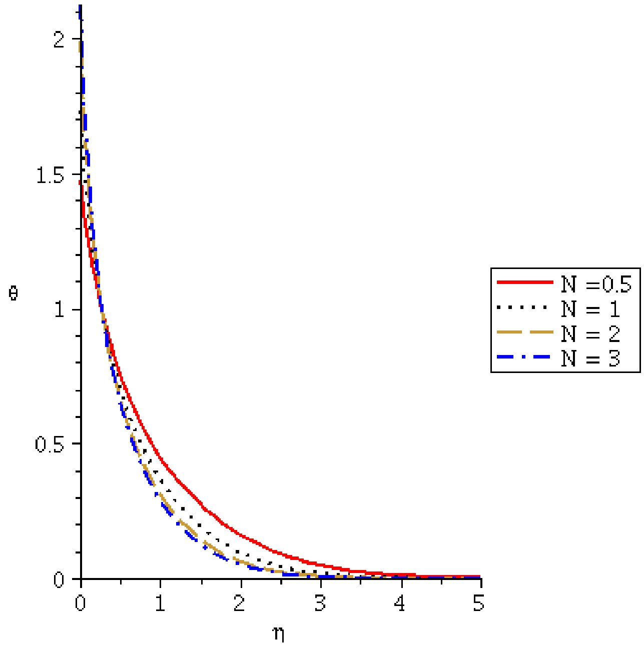

Figure 13: Temperature Profile with Increasing Radiation Parameter

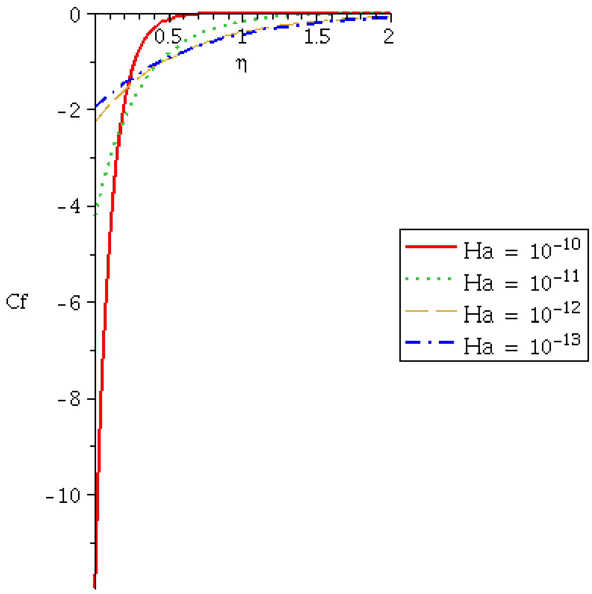

Figure 14: Skin Friction Coefficient Profile with Cu-Water Nanofluid for Decreasing Values Of Ha

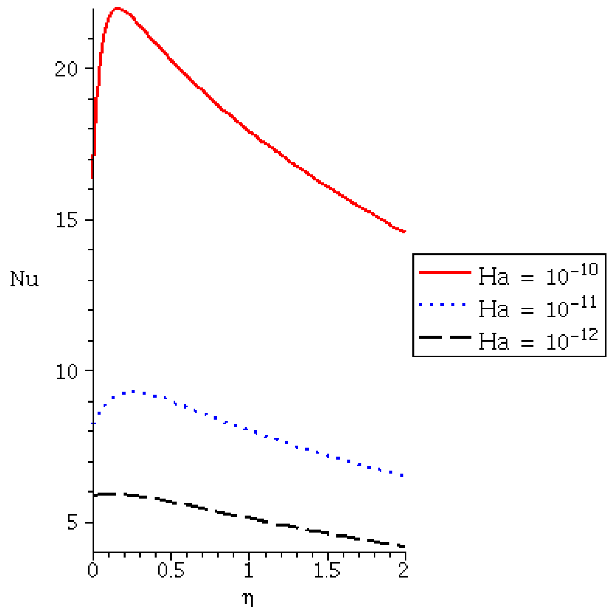

Figure 15: Nusselt Number Profile with Cu-Water Nanofluid for Decreasing Values of Ha

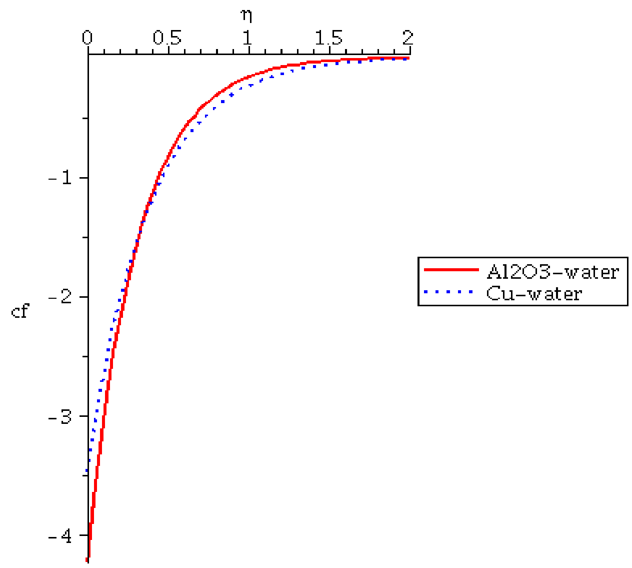

Figure 16: Skin Friction Coefficient Profile for Different Nanofluid

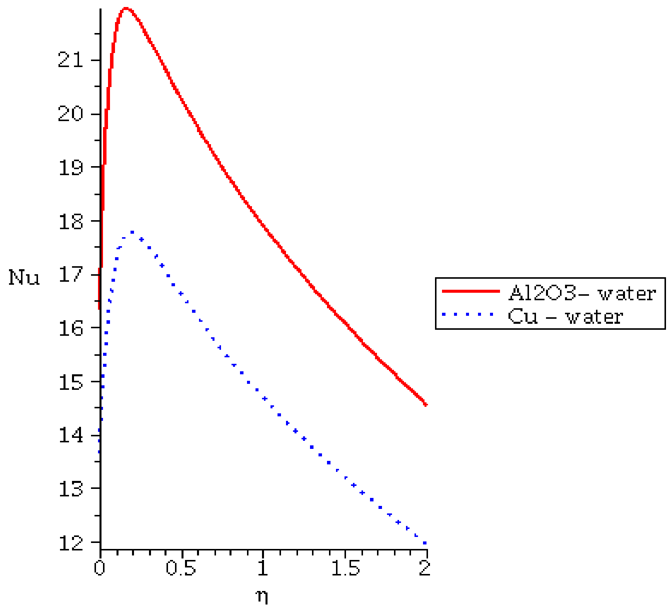

Figure 17: Nusselt Number Profile for Different Nanofluid

DISCUSSION OF RESULTS

Figure. (1) shows the effect of Hartmann number on the velocity profile and we observe that the decrease in Hartmann number decreases the velocity profile. Fig. (2) shows that a decrease in Hartmann number effect a corresponding decrease in the temperature profile. Fig. (3) shows that the increase in the Suction Parameter leads to a decrease in the temperature profile. Fig. (4) shows that an increase in the suction parameter leads to a decrease in the velocity profile. Fig. (5) shows that Alumina water is higher than the Copper water in the velocity profile. Fig. (6) shows that Copper water is lower than Alumina water in the temperature profile. Fig. (7) shows that the increase in solid volume parameter leads to an increase in the temperature profile. Fig. (8) shows that an increase in solid volume parameter leads to a decrease in velocity profile. Fig. (9) shows that an increase in Brinkmann number effects increase in the temperature profile. Fig. (10) shows that an increase in Biot number leads to an initial decrease in temperature profile but switches to increase. Fig. (11) shows that an increase in permeability parameter leads to a decrease in the temperature profile. Fig. (12) shows that the increase in permeability parameter leads to an increase in the velocity profile. Fig. (13) shows that the increase in the radiation parameter leads to a decrease in the temperature profile. Fig. (14) shows that as Ha decreases the skin friction coefficient profile increases but switches to decrease around 0.3. Fig. (15) shows that as the Ha decreases, the Nusselt number Profile increases a little and then decreases. Fig. (16) shows that Alumina water is initially smaller but at about 0.4 became higher on Skin Friction coefficient profile. Fig. (17) shows that Alumina water is higher than copper water on the Nusselt Number Profile.

CONCLUSIONS

The combined effects of thermal radiation and Newtonian heating on hydromagnetic boundary layer flow of nanofluids past a permeable moving surface were investigated and the following conclusions were made As Hartman number (Ha), Solid volume parameter (φ) and Suction parameter (S) increase, the momentum boundary layer thickness decreases but increases as the Permeability parameter (κ) increases. As Hartmann number (Ha) and solid volume parameter (φ) increase, the thermal boundary layer thickness increases while it decreases as the Suction parameter (S) and

Permeability parameter (κ) increase. As the Biotnumber (Bi) and Brinkmann number (Br) increase, we observe a corresponding increase in the thermal boundary layer With an increase in the thermal radiation Parameter(N), the thermal boundary layer starts increasing but switches to decrease after a certain point Cu-water showed a smaller momentum and thermal boundary layer thickness compared to Al2O3.

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